SOME RESULTS RELATED TO THEORY AND PRACTICE OF FUZZY CONCEPT LATTICES

Peter BUTKA

Technical University of Košice
Faculty of Electrical Engineering and Informatics
Department of Cybernetics and Artificial Intelligence
Letná 9, 040 01 Košice, Slovakia
peter.butka@tuke.sk

Abstract. This paper summarizes the work related to the theory and practical aspects of fuzzy concept lattices, mostly to generalized one-sided concept lattices. Concept lattice is hierarchical structure which organizes analysed objects into similar groups using the theory known as Formal Concept Analysis. During the latest work on the project we have analysed some new aspects related to these topics. In theoretical contemplations we have studied relations between fuzzy concept lattices and classical framework for concept lattices. In more practical studies we have analysed interpretation and usage of models based on the generalized one-sided concept lattices, as well as some of its implementation aspects. In this paper short summary of papers on several studied aspects is presented with the necessary references to their original versions.

1. Introduction

One of the conceptual data mining method is called Formal Concept Analysis (FCA, [8]), which is a theory of data analysis for identification of conceptual structures among data sets and was successfully applied in different data/text mining tasks. FCA produces concept lattice that can be understand as knowledge-based model or concept hierarchy. Standard FCA works in crisp case based on binary input data tables (object has/has not attribute). The fuzzy generalization based on the Galois connections was presented in [10] (another extension in [11]). This approach was used as a basic for the development of our model known as Generalized One-Sided Concept Lattices (GOSCL) [9], which is able to work with the input data table containing different types of attributes (binary, ordinal, nominal, real-valued, etc.). In the next chapters, several papers related to fuzzy concept lattices (and most of them to GOSCL) are described.

2. Theoretical papers

From our latest works, several of them were related to the theoretical details of fuzzy concept lattices.

In [1] we have provided theoretical details regarding the possibility to represent fuzzy concept lattices, defined via antitone Galois connections, within the framework of classical FCA. One important fact is that all needed information is explicitly contained in a given formal fuzzy context and the proposed representation can be extracted without a creation of the corresponding fuzzy concept lattice. This representation transforms a fuzzy formal context into a binary formal context. As it was shown, this transformation maintains all the information given by the lattice structure of a concept lattice, since the corresponding concept lattices are isomorphic. Consequently, the well developed theory of classical FCA can be used for studying the fuzzy concept lattices or an arbitrary algorithm for classical concept lattices can be used for the creation of the fuzzy concept lattices.

In [2] we have analysed the relationship between the methods of conceptual scaling and generalized one-sided concept lattices (GOSCL). These both methods represent different possibilities on how to deal with many-valued contexts. Within the paper we briefly described these methods and prove that they are equivalent according to fact, that their closure systems are isomorphic. It means that we show that the application of these two approaches to a given many-valued context yields the same closure system on the set of all objects. Also, based on this equivalence, we proposed a possible attribute reduction of one-sided formal contexts. Therefore, both methods can be used and combined for practical tasks or for research of similar relevant approaches.

In [2] we have provided the proof of assertion which is called Basic theorem for generalized one-sided concept lattices. It is basic assertion (tool) for a theoretical study of GOSCL-like models. It is similar assertion to one known from classical framework known as Basic Theorem for Concept Lattices.

3. Studies on practical aspects of interpretation and usage of GOSCL

During latest period we have also analysed some practical and implementation aspects of GOSCL model and its algorithm. In this chapter, these papers are shortly described.

In [4] we have analysed the possibilities for interpretation of fuzzy attribute subsets in GOSCL. In more details, we have analysed the problem how to reduce number of attributes within the existing model and (according to this fact) how to increase interpretability of the FCA-based models. If we have original one-sided context, we are still able to analyse its subparts first. One of the way is to separate different types of attributes, but formally we can make division of attributes to disjoint subsets in any way. Then we are able to create concept lattice for every subpart (all of them are on same sets of objects) and use these concept lattices as truth value structures for finale merge of the concept lattice for the original context. The result is isomorphic lattice, but its attributes part has less number of elements equal to number of used subparts. Some other method for isomorphic result with specific change of attributes values is presented in [5], where separable fuzzy modifiers are used to find more convenient description of attributes of concepts within the concept lattice.

In [6] we have provided another interesting possibility for practical usage of GOSCL-based models. At the start we have introduced the basic representation of facets and their properties as knowledge representation model. Then, we have provided the model which is able to use GOSCL as facet-like representation, i.e., how fuzzy relations in facets are represented by relations in GOSCL. Moreover, GOSCL is able to find hierarchical structure of objects and their related attribute values, which leads to hierarchical facet-like structure. Additionally, we are able to provide also support for more complex structures using sub-models defining the nested line diagrams of modelled domain. All these aspects are also described by the set of illustrative examples.

In [7] we provide more implementation-based paper, which is related to the reduction of computation times of algorithm for creation of GOSCL models. We have used distribution-based algorithm based on the decomposition of input context to several subtables with the separated (and disjoint) subsets of rows, i.e., data table is decomposed to smaller tables (using recursive bisection-based method), for which small lattices are created and these are iteratively combined. Even if it is more effective for parallel/distributed computation, we have shown in several experiments that it is possible to get quite high reduction also for sequential run of such algorithm for very sparse domains. We have used generated data for the experiments with sparseness similar to textual datasets in order to prove this fact.

4. Conclusions

In this paper the summary of latest works related to the theory and applications of fuzzy concept lattices (and mostly generalized one-sided concept lattices) was presented. In the future we would like to follow the research and provide new theoretical and practical results in the area of formal concept analysis, with the application of such methods in data/text mining, information retrieval and business intelligence.

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